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Weighted Least Squares Method of Grid Generation

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Introduction

ONE of the important steps required to accurately solve a problem numerically is the generating of appropriate grids. To the authors' knowledge, the transfinite interpolation method[1,2] is the fastest of existing grid generation methods. However, if the resulting grid system cannot fulfill the users' requirement, one must redistribute the grid points on the boundaries or redefine the blending functions, which might take a long time for a complex domain. In this study, the authors propose the creation of a grid smoother to improve the local grid distortion, nonsmoothness, and overlapping of the transfinite interpolated grid points. The grid smoother, based on the weighted least square method,[3] is designed to cover either regular or irregular subregion(s) on the computational domain and to maintain the grid clustering or stretching characteristics of the original grid distribution.

Among the existing adaptive grid methods, the multiple one-dimensional adaptive grid method[4,5] is one of the fastest methods. Unfortunately, this method has the drawback of excessive grid distortion, even if the Jeng and Liou smoothing version[5] is applied. Therefore, the second purpose of this study is to smooth the resulting grid finding through the application of the Jeng and Liou modified multiple one-dimensional adaptive grid method.

Formulation

Suppose that we have an initial grid system, $(x^0_{i,j}, y^0_{i,j})$, which has some grid imperfections. First, define the two-dimensional grid functional to be

$$F = \sum_{i,j} q_{i,j} \left\{ \sum_{k=\pm 1} \left[\frac{(x_{i+k,j} - x_{i,j})^2 + (y_{i+k,j} - y_{i,j})^2}{\sqrt{\left(x^0_{i+k,j} - x^0_{i,j}\right)^2 + \left(y^0_{i+k,j} - y^0_{i,j}\right)^2}} + \frac{(x_{i,j+k} - x_{i,j})^2 + (y_{i,j+k} - y_{i,j})^2}{\sqrt{\left(x^0_{i,j+k} - x^0_{i,j}\right)^2 + \left(y^0_{i,j+k} - y^0_{i,j}\right)^2}} \right] + 2\nu \left[\frac{(x_{i+1,j} - 2x_{i,j} + x_{i-1,j})^2 + (y_{i+1,j} - 2y_{i,j} + y_{i-1,j})^2}{\sum_{k=\pm 1} \sqrt{\left(x^0_{i+k,j} - x^0_{i,j}\right)^2 + \left(y^0_{i+k,j} - y^0_{i,j}\right)^2}} + \frac{(x_{i,j+1} - 2x_{i,j} + x_{i,j-1})^2 + (y_{i,j+1} - 2y_{i,j} + y_{i,j-1})^2}{\sum_{k=\pm 1} \sqrt{\left(x^0_{i,j+k} - x^0_{i,j}\right)^2 + \left(y^0_{i,j+k} - y^0_{i,j}\right)^2}} \right] \right\} \tag{1}$$

where the denominators serve as weighting functions, $q_{i,j}$ denotes the localized switching function, and ν is a user specified parameter. The weighting function takes the first power of the grid spacing to ensure grid smoothness in case the initial grids are unequally spaced. By taking the first-order partial derivatives of F with respect to $x_{i,j}$ and $y_{i,j}$ separately, and by setting these derivatives to be zero, the linear grid equations for the x coordinate are easily formulated.

$$a_2 x_{i+2,j} + a_1 x_{i+1,j} + a_0 x_{i,j} + a_{-1,j} x_{i-1,j} + a_{-2} x_{i-2,j} + b_2 x_{i,j+2} + b_1 x_{i,j+1} + b_{-1} x_{i,j-1} + b_{-2} x_{i,j-1} = 0 \tag{2}$$

where the coefficients a and b are functions of the initial coordinates ($x^0_{i,j}$ and $y^0_{i,j}$) and equations for the y coordinate are in a similar form with x replaced by y. Note that in these final grid equations, the terms of the grid spacings of Eq. (1) correspond conceptually to the terms $x_{\xi\xi}, x_{\eta\eta}, \ldots$, of the elliptic equation method, whereas terms of the second-order differencing conceptually correspond to the fourth order smoothing terms $x_{\xi\xi\xi\xi}, \ldots$.

If $q_{i,j}$ of Eq. (1) is replaced by the unit value, the method can be applied to the entire grid. In general, the transfinite interpolation method generates smooth grid points over the whole computational domain except at some isolated regions. Therefore, it is reasonable to apply the method of Eqs. (1) and (2) only at points around these isolated regions by properly choosing $q_{i,j}$. The switching functions can be defined in different ways. For example, if grid overlapping is to be avoided, they are defined as

$$q_{i,j} = \text{integer}\left| \frac{|J_{i,j}| - J_{i,j} + 2\epsilon}{2|J_{i,j}| + \epsilon} \right| \tag{3}$$

where ϵ is a small positive parameter which prevents dividing by zero, and $J = x_\xi y_\eta - x_\eta y_\xi$. Equation (3) returns a zero value of $q_{i,j}$ for $J > 0$ and an unit value for $J \leq 0$.

To obtain a smooth transition between the adjusting region and that without adjusting, every adjusting region where $q = 1$ should contain a transition margin enclosing the unsatisfactory grid points. Therefore, the following extension is performed twice:

$$t_{i,j} = q_{i-1,j-1} + q_{i,j-1} + q_{i+1,j-1} + q_{i-1,j} + q_{i,j} + q_{i+1,j} + q_{i-1,j+1} + q_{i,j+1} + q_{i+1,j+1} \tag{4}$$

where $q = 0$ or 1, and then

$$q_{i,j} = \text{integer}\left| \frac{s_{i,j} + 1}{2} \right|, \qquad s_{i,j} = \text{Sign}(1., t_{i,j} - 0.5) \tag{5}$$

where the Sign symbol denotes a Fortran function such that $s_{i,j} = 1$ if $t_{i,j} \geq 1$ and otherwise equals 0. If the resulting grid distribution is not satisfactory, the extension can be repeatedly performed. By substituting these switching functions into Eq. (1) and solving Eq. (1) in terms of the point Gauss-Seidel iteration, the present method can be applied to every arbitrary subregion of the computational domain.

Results and Discussions

The linear transfinite interpolation method,[2] whose independent variables are ξ and η, is first applied to a simple geometry to construct an algebraic grid system (20×20 points). To demonstrate the characteristics of the initial grid system, assume that grid clustering is desired at the center region. Since the grid point adjustment along the boundaries is purposly made improper, grid overlapping is present at the center region as shown in Fig. 1. After employing the present local adjusting method to improve the undesired grid overlapping and to maintain the grid clustering, the result is satisfactory, as shown in Fig. 2. The CPU time required is 6.4 s on an HP720 workstation, which is significantly less than the 18.8 s required if the entire grid is adjusted.

The second example is an inviscid transonic flow over a 10% bump, where the inflow Mach number takes a value of 0.675. The initial grid found by the linear transfinite interpolation method has a grid size of 61×17 points. Because of the space limitation, both the initial grid system and the initial solutions calculated by the Harten-Yee minmod total variation diminishing (TVD) scheme[6] are not shown. Then, the application of the Jeng and Liou[5] multiple one-dimensional adaptive grid scheme (with $\lambda_\xi = 2$ and $\lambda_\eta = 0.5$, where ξ = const is a vertical grid line) gives the grid distribution of Fig. 3. Although the multiple one-dimensional adaptive grid scheme is very

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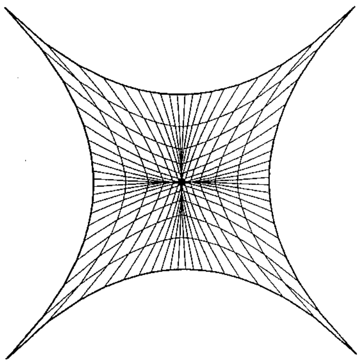

Fig. 1 Initial grid distribution generated by the linear transfinite interpolation method, 20 × 20 points, in a simple region with convex boundaries.

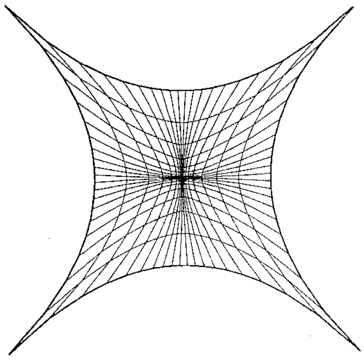

Fig. 2 Grid distribution generated by the present local grid smoothing method, ν = 10, using Fig. 1 as the initial grid system.

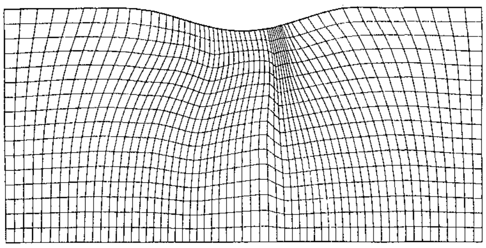

Fig. 3 Adaptive grid distribution for flow over a channel with a 10% bump, generated by the Jeng and Liou multiple one-dimensional adaptive grid scheme, 61 × 17 points, as initial data.

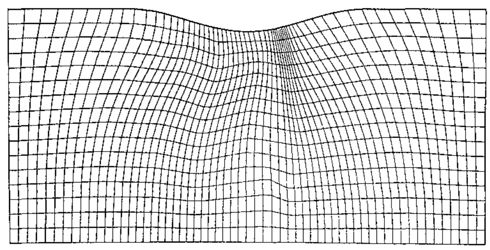

Fig. 4 Adaptive grid distribution for flow over a channel with a 10% bump, improved by the present local grid smoothing method, using Fig. 3 as the initial grid system.

fast (less than 1 s on an HP720 workstation), the grid distribution across the shock is not smooth enough, which causes the constant Mach lines to have small wiggles (not shown here) before the shock.

Instead of using the Jacobian in Eq. (3), the criterion employed here is that the intersection angle between successive grid lines should be larger than 1/10 of the maximal intersection angle of the initial grid system. The resulting grids are shown in Fig. 4, where the nonsmooothness of Fig. 3 across the shock is properly smeared. For the sake of brevity, the physical solution found by the minmod TVD scheme is not shown. Note that the Mach contours of the solution coincide with those employing the present method in the entire domain and employing the Anderson adaptive grid scheme[7,8] very well. The required CPU time for generating the adaptive grid systems of these methods are 11.8, 126.6, and 351.1 s, respectively, which demonstrates the usefulness of the present method in solving this simple problem. However, for a complex flowfield, the adaptive grid system generated by the multiple one-dimensional adaptive grid scheme may be unsatisfactory throughout the whole computing domain, and the present method can not be applied.

Conclusions

The weighted least squares method is successfully developed as a local grid smoother. The method needs an initial grid system, which is supported by fast grid schemes such as the transfinite interpolation method or the multiple one-dimensional adaptive grid scheme. Numerical evidence illustrates that the proposed method properly smoothes out the initial grids and preserves the initial grid clustering and stretching. Therefore, the applicability of these two fast schemes is greatly improved.

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Jet Mixing Control Using Excitation from Miniature Oscillating Jets

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Introduction

THE viability and performance of an unsteady fluid-dynamic excitation system were investigated. The objective was to develop and successfully demonstrate the use of excitation devices for the

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